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Applied alternating method to analyze the stress concentration around interacting multiple circular holes in an infinite domain

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Abstract

This paper presents an efficient alternating method for analyzing the interactions among multiple circular holes in a two-dimensional infinite domain. An analytical solution is derived for a single circular hole in an infinite domain subjected to the arbitrary tractions across the circle boundary to achieve this purpose. This analytical solution correlates with a successive iterative superposition process capable of satisfying the prescribed boundary condition for each circular hole of the problem. In addition, several perforated plate problems are solved to demonstrate the proposed method's validity. The computed results and the available referenced solutions closely corresponds to each other and indicates the method's accuracy and efficiency.

1. Introduction

In engineering, solving the elastic plane problems for a plate with multiple holes is of considerable interest. The reliable evaluation for reducing the accompanying stress concentration factor around these holes is necessary to assure the structural integrity of the perforated plate.

A series of analytic techniques have been devoted to treat the problem of the interacting holes in infinite plate. Early attempts to solve these problems were restricted to two (see, for example, Ling, 1948; Atsumi, 1956; Haddon, 1967; Kienzler and Zhuping, 1987; Greenwood, 1989) or periodic set (see, for example, Isida and Igawa, 1991; Meguid et al. 1996) of equal holes. In addition, Horii and Nemat-Nasser (1985) and Meguid and Shen (1992) used the complex potentials

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and the appropriate superposition procedure to formulate the approximate solutions for the calculation of the stress field in a solid containing any number of holes. However, these investigations were restricted to the coaxial distributions of holes of various sizes or random distributions of equal holes in infinite domain. Bird and Steel (1991, 1992) used the Fourier series procedure to solve the two-dimensional bi-harmonic and harmonic equations on multiply connected circular domains. The number of studies for the elastic problems of multiple holes with arbitrary distributions and various sizes are still limited.

Recently, the alternating method (Kantorovich and Krylov, 1964) has been successfully used to analyze a number of two-dimensional and antiplane fracture problems with multiple cracks arbitrarily distributed in infinite domains (Chen and Chang, 1990; Ting et al., 1994, 1995). Nevertheless, this efficient procedure has paid little attention to the issue of the perforated plates. Thus, the extension of the previous works to analyze the interactions among the arbitrary distribution of multiple circular holes with various sizes is very interesting, and is the main objective of this work.

In the process of the alternating method, an analytical solution of the single circular hole in the infinite domain subjected to the arbitrary surface tractions over the circumference of the hole must be obtained. Although this analytic solution has been solved by the Muskhelishvili (1953) complex method in general forms, the elastic solutions corresponding to the arbitrary tractions expressed by the traditional trigonometric Fourier series are completely derived using Airy stress function for the purposes of simplicity and efficiency. These analytical solutions are then used for the successive iterative superposition procedures to satisfy the prescribed boundary conditions.

The validity of the present work is verified by analyzing several problems of multiple holes with different sizes and arrangements. The interactions among holes are studied in detail. The results were compared with other reference solutions to show that with simple technique and minimum computing effort they are capable of obtaining accurate stress concentration factor.

2. Analytical solution for a hole with arbitrary surface tractions in an infinite domain

Figure 1 shows an infinite domain with a circular hole of radius R and center at $X = 0$ and $Y = 0$. The hole surface is subjected to an arbitrary normal force $T_r(\theta)$ and shear force $T_\theta(\theta)$. For the plane stress problem neglecting the body forces, the stresses are derived from Airy stress function Φ satisfying the bi-harmonic equation

$$\nabla^4\Phi = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\Phi = 0$$

The stress components relative to the polar coordinates are

$$\sigma_r = r^{-1}\Phi_{,r} + r^{-2}\Phi_{,\theta\theta}, \quad \sigma_\theta = \Phi_{,rr}, \quad \tau_{r\theta} = -(r^{-1}\Phi_{,\theta})_{,r}$$

The radial and tangential loads on the hole surface can be expressed by the trigonometric Fourier series as

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Fig. 1. Arbitrary normal and shear load applied to the hole boundary in an infinite domain.

$$T_r(\theta) = \sum_{m=0}^{n} a_m \cos(m\theta) + \sum_{m=1}^{n} b_m \sin(m\theta), \quad T_\theta(\theta) = \sum_{m=0}^{n} c_m \cos(m\theta) + \sum_{m=1}^{n} d_m \sin(m\theta)$$

The Airy stress function can be assumed as

$$\begin{aligned}\Phi = {} & (a_0+c_0)(f_1 \ln r + f_2 r^2 \ln r + f_3 r^2) + c_0(f_4 r \sin\theta + f_5 r\cos\theta + f_6\theta) \\ & + (f_7 r^3 + f_8 r + f_9 r^{-1} + f_{10} r \ln r)[(a_1+d_1)\cos\theta + (b_1+c_1)\sin\theta] \\ & + f_{11} r\theta[(a_1+d_1)\sin\theta + (b_1+c_1)\cos\theta] \\ & + \sum_{m=2}^{n} [f_{12} r^{m+2} + f_{13} r^m + f_{14} r^{-m} + f_{15} r^{-(m-2)}][(a_m+d_m)\cos(m\theta) + (b_m+c_m)\sin(m\theta)]\end{aligned}$$

where f_i are undetermined constants. Using the boundary conditions $\sigma_r = -T_r$ or $\tau_{r\theta} = -T_\theta$ at $r = R$ and the stresses approach zero as r tend to infinity, and the compatibility conditions of the displacement components, the stress and displacement components can then be completely derived as follows:

$$\begin{aligned}\sigma_r = {} & -a_0\left(\frac{R}{r}\right)^2 - \left[A\left(\frac{R}{r}\right)^3 - (A-1)\left(\frac{R}{r}\right)\right][a_1\cos\theta + b_1\sin\theta] \\ & + \left[B\left(\frac{R}{r}\right)^3 + (A-1)\left(\frac{R}{r}\right)\right][c_1\sin\theta - d_1\cos\theta] \\ & + \sum_{m=2}^{n}\left[\frac{m}{2}\left(\frac{R}{r}\right)^{m+2} - \frac{m+2}{2}\left(\frac{R}{r}\right)^{m}\right][a_m\cos(m\theta) + b_m\sin(m\theta)] \\ & + \sum_{m=2}^{n}\left[\frac{m+2}{2}\left(\frac{R}{r}\right)^{m+2} - \frac{m+2}{2}\left(\frac{R}{r}\right)^{m}\right][c_m\sin(m\theta) - d_m\cos(m\theta)]\end{aligned}$$

$$\sigma_\theta = a_0\left(\frac{R}{r}\right)^2 + A\left[\left(\frac{R}{r}\right)^3 + \left(\frac{R}{r}\right)\right][a_1\cos\theta + b_1\sin\theta] + \left[-B\left(\frac{R}{r}\right)^3 + A\left(\frac{R}{r}\right)\right][c_1\sin\theta - d_1\cos\theta]$$

$$+\sum_{m=2}^{n}\left[-\frac{m}{2}\left(\frac{R}{r}\right)^{m+2} + \frac{m-2}{2}\left(\frac{R}{r}\right)^{m}\right][a_m\cos(m\theta) + b_m\sin(m\theta)]$$

$$+\sum_{m=2}^{n}\left[-\frac{m+2}{2}\left(\frac{R}{r}\right)^{m+2} + \frac{m-2}{2}\left(\frac{R}{r}\right)^{m}\right][c_m\sin(m\theta) - d_m\cos(m\theta)]$$

$$\tau_{r\theta} = -c_0\left(\frac{R}{r}\right)^2 + A\left[-\left(\frac{R}{r}\right)^3 + \left(\frac{R}{r}\right)\right][a_1\sin\theta - b_1\cos\theta]$$

$$+\left[-B\left(\frac{R}{r}\right)^3 - A\left(\frac{R}{r}\right)\right][c_1\cos\theta + d_1\sin\theta]$$

$$+\sum_{m=2}^{n}\left[\frac{m}{2}\left(\frac{R}{r}\right)^{m+2} - \frac{m}{2}\left(\frac{R}{r}\right)^{m}\right][a_m\sin(m\theta) - b_m\cos(m\theta)]$$

$$+\sum_{m=2}^{n}\left[-\frac{m+2}{2}\left(\frac{R}{r}\right)^{m+2} + \frac{m}{2}\left(\frac{R}{r}\right)^{m}\right][c_m\cos(m\theta) + d_m\sin(m\theta)]$$

$$u_r = C\frac{r}{E}\left(\frac{R}{r}\right)^2 a_0 + \frac{r}{E}\left[D\left(\frac{R}{r}\right)^3 + (F-1)\left(\frac{R}{r}\right)\ln r\right](a_1\cos\theta + b_1\sin\theta)$$

$$+\frac{r}{E}\left[-G\left(\frac{R}{r}\right)^3 + (F-1)\left(\frac{R}{r}\right)\ln r\right]$$

$$\times(c_1\sin\theta - d_1\cos\theta) + \sum_{m=2}^{n}\frac{r}{E}\left[\frac{-m}{2(m+1)}C\left(\frac{R}{r}\right)^{m+2}\right.$$

$$\left. + \frac{(m+2)+v(m-2)}{2(m-1)}\left(\frac{R}{r}\right)^{m}\right][a_m\cos(m\theta) + b_m\sin(m\theta)]$$

$$+\sum_{m=2}^{n}\frac{r}{E}\left[\frac{-(m+2)}{2(m+1)}C\left(\frac{R}{r}\right)^{m+2} + \frac{(m+2)+v(m-2)}{2(m-1)}\left(\frac{R}{r}\right)^{m}\right][c_m\sin(m\theta) - d_m\cos(m\theta)]$$

$$u_\theta = C\frac{r}{E}\left(\frac{R}{r}\right)^2 c_0 + \frac{r}{E}\left[D\left(\frac{R}{r}\right)^3 + (F - F\ln r + \ln r + v)\left(\frac{R}{r}\right)\right](a_1\sin\theta - b_1\cos\theta)$$

$$+\frac{r}{E}\left[G\left(\frac{R}{r}\right)^3 - (F - F\ln r + \ln r + v)\left(\frac{R}{r}\right)\right](c_1\cos\theta + d_1\sin\theta)$$

$$+\sum_{m=2}^{n}\frac{r}{E}\left[\frac{-m}{2(m+1)}C\left(\frac{R}{r}\right)^{m+2} + \frac{(m-4)+vm}{2(m-1)}\left(\frac{R}{r}\right)^{m}\right][a_m\sin(m\theta) - b_m\cos(m\theta)]$$

$$+\sum_{m=2}^{n}\frac{r}{E}\left[\frac{(m+2)}{2(m+1)}C\left(\frac{R}{r}\right)^{m+2}-\frac{(m-4)+vm}{2(m-1)}\left(\frac{R}{r}\right)^{m}\right][c_m\cos(m\theta)+d_m\sin(m\theta)]$$

where $A=(1-v)/4$, $B=(3+v)/4$, $C=1+v$, $D=(1-v^2)/8$, $F=(1-v)^2/4$, $G=(3+v)(1+v)/8$, E is Young's modulus and v is the Poisson's ratio.

3. Alternating method solving multiple holes in an infinite domain

Consider a two-dimensional infinite solid containing k circular holes under farfield uniform stresses, as shown in Fig. 2(a). This problem can be expressed as a superposition of two cases, as illustrated in Fig. 2(b) and 2(c). Figure 2(b) represents the case when external forces are acting on the infinite plane without any holes and Fig. 2(c) the case with multiple circular holes under the fictitious tractions along the hole boundaries. Hence, based on the analytical solution of a circular hole under arbitrary surface tractions in an infinite plate derived in the previous section, the iterative processes of the alternating method, illustrated in Fig. 3, can be stated as follows:

(1) Consider the arbitrary load distributions T_j^i on the boundary surface of the circular hole j ($j=1,2,\ldots,k$) for the iterative cycle i by Fourier series represented, as shown in Fig. 3(a). T_j^i can be written in matrix form as

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Fig. 2. Superposition scheme for the alternating method.

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Fig. 3. Alternating procedures.

$$\{T\}_j^i = \begin{Bmatrix} T_r \\ T_\theta \end{Bmatrix}_j^i = [A]_j^i \cdot \{P\}$$

and

$$[A] = \begin{bmatrix} a_0 & a_1 & a_2 & \dots & a_n & b_1 & b_2 & \dots & b_n \\ c_0 & c_1 & c_2 & \dots & c_n & d_1 & d_2 & \dots & d_n \end{bmatrix}$$

$$\{P\} = \{1 \quad \cos\theta \quad \cos 2\theta \quad \dots \quad \cos n\theta \quad \sin\theta \quad \sin 2\theta \quad \dots \quad \sin n\theta\}^T$$

where the matrix $[A]_j^i$ consists of the coefficients of Fourier series for the jth hole for the ith iteration. The superscript T designated the transpose of the matrix.

(2) Using the analytical solution obtained in the previous section, the interactive residual tractions t_{1n}^i induced by hole 1 at an imaginary hole n ($n = 2, 3, \dots, k$) can be computed (see Fig. 3(b)).

Thus, the original loading force T_2^i of the hole 2 (shown in Fig. 3(c)) must minus the interactive residual tractions t_{12}^i that is the influence of the hole 2 induced by hole 1, i.e., $T_2^i - t_{12}^i$. And the residual tractions of the other holes corresponding to these residual tractions are t_{2n}^i $(n = 1, 3, 4, \ldots, k)$. In general, the residual tractions for hole k is $T_k^i - t_{1k}^i - t_{2k}^i - \cdots - t_{(k-1)k}^i$, and the corresponding coefficient matrix is

$$[A]_k^i - [A]_{1k}^i - [A]_{2k}^i - \cdots - [A]_{(k-1)k}^i$$

The superposition procedure can be repeated with the next residual loads T_j^{i+1} for the iterative cycle $i+1$, as shown in Fig. 3(f). The residual loads and the corresponding coefficient matrix will be reduced after each cycle of the iteration and may be expressed as

$$T_j^{i+1} = -\sum_{n=j+1}^{k} t_{nj}^i, \quad [A]_j^{i+1} = -\sum_{n=j+1}^{k} [A]_{nj}^i, \quad j = 1, 2, \ldots, k-1$$

(3) Once the ratio of residual stresses is released/permissible stress for each hole becomes smaller than a small value γ, say

$$\sum_{j=1}^{k} |T_j^{i+1}|/S < \gamma, \quad \gamma = 0.001$$

Here S is the initial stresses applied on hole surfaces. When the iteration is finished, step 5 is to be executed. If the tractions are not negligible, the iteration is not complete and therefore goes to step 4.

(4) Repeat all the iteration procedures (i.e. step 1 to step 3) until the residual tractions on each hole are negligible, considering the residual tractions of step 2 as newly applied loading acting on each hole surface.

(5) Evaluate the final stress components and stress concentration factor.

4. Results and discussion

A novel alternating method is proposed in this study, the validity of which was tested using a number of examples of an infinite domain with two or multiple circular holes with various sizes and arbitrary distributions. The solutions developed are compared with the works of Haddon (1967), Horii and Nemat-Nasser (1985) and Meguid and Shen (1992). Also, the effects of the interactions among multiple holes on the stress concentration factor at the boundaries of the circular holes were studied and presented.

4.1. *Two circular holes in an infinite domain*

Problems of an infinite domain with two equal or unequal co-axial circular holes are shown in the upper right corner of Figs 4–10, under farfield uniform stresses σ. The holes 1 and 2 with radii a_1 and a_2 are named as main and defense holes, respectively. The origin of global axes X and Y is at the center of the main hole 1. The inclination of the defense hole corresponding to the main

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Fig. 4. The variation of the normalized tangential stress at the boundary of the main hole as a function of θ for different proportionality constants α with the two equal holes configuration ($\beta = 0^\circ$).

hole is β. The distance between the centers of the two holes is $a_1 + a_2 + \alpha a_1$, where α is the proportionality constant. The effects of two interacting holes on the stress distributions as a function of α and β are discussed as follows.

(1) First, two holes of equal sizes in an infinite domain were considered. The farfield simple tension is applied in the Y-direction. The interaction between two holes was studied using five sets of α with values 0.2, 0.4, 1.0, 4.0 and 10.0. Figures 4 and 5 depict the computed normalized tangential stresses σ_θ/σ along the circular boundary for two angles of inclination $\beta = 0^\circ$ (i.e., two collinear holes normal to the loading direction) and $\beta = 90^\circ$ (i.e., two collinear holes along the loading direction). The results of the present alternating method are in close agreement with the reference solution of Horii's (1985), as evident in the two figures. The maximum and minimum normalized tangential stresses, and the corresponding locations θ (in degrees) are listed in Table 1. Again, the values are consistent with those of Haddon (1967). When two holes are nearly close to each other (i.e. $\alpha = 0.2$), the main hole was found to have a larger normalized tangential stress, namely 6.107, when the loading is normal to the two collinear holes. The peak value indicates that when the holes are very close, their interactive effects are strong. When two holes become separated by an appropriate distance (i.e. $\alpha = 10.0$), the results for $\beta = 0^\circ$ and $\beta = 90^\circ$ were observed to have almost the same value. This phenomenon suggests that the interactive effect between the two holes is insignificant when $\alpha = 10.0$. Figure 6 illustrates the calculated stress concentration factor (SCF),

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Fig. 5. The variation of the normalized tangential stress at the boundary of the main hole as a function of θ for different proportionality constants α with the two equal holes configuration ($\beta = 90^\circ$).

the maximum normalized tangential stress σ_θ/σ along the boundary of hole, vs the inclination β of the defense hole to the main hole, corresponding to each value of α chosen for the present study. Superposed on this figure are the solution given in Meguid and Shen (1992) for $\alpha = 1.0$. The maximum and minimum of SCF, for the inclinations β_1 and β_2 of the defense hole to the main hole, for each value of α are listed in Table 2. The closer the two holes are, the larger the normalized tangential stresses are. If the inclination is in some range, SCF can be less than the stress concentration factor $k_0 = 3.0$ for a circular hole in infinite domain (Timoshenko and Goodier, 1970). These ranges of inclination are also shown in Table 2.

(2) Two collinear holes of unequal size in an infinite domain when subjected to a normal farfield loading and a parallel one, i.e. $\beta = 0^\circ$ and $\beta = 90^\circ$, are studied. The stress concentration factors of the main hole are affected by five different defense hole sizes, $0.2a_1$, $0.4a_1$, $0.6a_1$, $0.8a_1$ and $1.0a_1$, as functions of the proportionality constants α and are depicted in Figs 7–10. The curves for the size of defense hole $0.2a_1$ vs $\alpha = 0.2$, 0.8 and 2.0 shows perfect agreement with those of Horii and Nemat-Nasser (1985). Consider the case $\beta = 0^\circ$; Fig. 7 suggests that the increase in the size of the defense hole would result in the increase of the stress concentration factor of the main hole. Figure 8 indicates that SCF of the defense hole is independent of the hole size. Furthermore, the role of

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Fig. 6. The variation of SCF at the main hole as a function of inclination angle β for different proportionality constants α with the two equal holes configuration.

the defense hole becomes insignificant (i.e. $|\mathrm{SCF}-k_0|/k_0 < 0.05$) if the proportionality constant α exceeds 1.3. The phenomenon of SCF of the main hole in the case of $\beta = 90^\circ$ is contrary to that of $\beta = 0^\circ$. Figure 9 suggests that the increase in the size of the defense hole reduces SCF at the main hole. The interaction between the main hole and the defense hole is irrelevant when α exceeds 4.6.

4.2. *Three collinear circular holes in an infinite domain*

The upper panel of Fig. 11 illustrates the case of an infinite domain with three equal or unequal collinear circular holes, under farfield uniform stress σ in the Y-direction. The central hole 1 is denoted as a main hole and the others are defense holes with radii a_1 and a_2, respectively. The origin of global axes X and Y is at the center of the main hole 1. The inclination of the defense hole to the main hole is β. The distance between the centers of the two holes is $a_1+a_2+\alpha a_1$. Following is a discussion of the iterations among multiple holes on the stress distributions as a function of α and β.

Consider five different distances αa_1, with $\alpha = 0.2$, 0.4, 1.0, 4.0 and 10.0, among three equal

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Fig. 7. SCF at the main hole as a function of α corresponding to different sizes of defense hole for two holes configuration ($\beta = 0^\circ$).

collinear circular holes. The variations of SCF at the main and defense holes with the inclination angle β are illustrated in Figs 11 and 12. The results of SCF at the main and defense holes for $\beta = 0^\circ$ and $\beta = 90^\circ$, and for the center distance $3a_1$ calculated by Meguid and Shen (1992) fits perfectly to the curves obtained from our calculation. Those values of β_1 and β_2 for the main and defense holes which make SCF maximum and minimum, respectively, are listed in Table 3. For the main hole, the variations of SCF vs the inclination β from 75–105° are not significant.

For different sizes of the defense hole, the variations of SCF with the distance between the centers of the holes subjected to the farfield loads normal and parallel to three collinear holes are depicted in Figs 13–16. These curves closely resemble those of two unequal holes as shown in Figs 7–10. In the case of $\beta = 0^\circ$, SCF of three holes is higher than those of two holes due to the effect of a higher number of holes. If the size of the defense hole is very small compared with the main hole, the effect of the defense hole cannot be reached even though the three holes are very close. On the contrary, when the holes are located parallel to loading (i.e. $\beta = 90^\circ$), the SCF of three holes is smaller than that of two holes. On the other hand, the interactions between the main hole and the defense hole shown in Figs 13 and 15 becomes insignificant when α exceeds 1.7 and 7.9,

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Fig. 8. SCF at the defense hole as a function of α corresponding to different sizes of defense hole for two holes configuration ($\beta = 0°$).

respectively. The results of the proposed method are in good agreement with the reference solution for three equal holes calculated by Meguid and Shen (1992).

4.3. *Multiple holes in an infinite domain*

Consider a square arrangement with a central hole. Five different ratios of the sizes of the main and the defense holes, namely, $a_2/a_1 = 1.0, 0.8, 0.6, 0.4, 0.2$ and $a_2 = a_3 = a_4 = a_5$, were selected to study the interactions among holes. Figures 17 and 18 show the variation of SCF at the main hole and the defense hole, respectively, with the proportionality constant α corresponding to the five size ratios. The reference solution of Meguid and Shen (1992) for the case $\alpha = 1.0$ and $a_2/a_1 = 1$ is in agreement with the present results. These figures show that the SCF at the main hole increases with the increase in the size of the defense hole. However, if the value of α is larger than 8.2, the effects of the defense holes can be neglected.

5. Conclusions

In this work, the general analytical solution for arbitrary radial and tangential force distributions on the surface of the hole in an infinite domain is successfully derived. Based on this solution, the

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Fig. 9. SCF at the main hole as a function of α corresponding to different sizes of defense hole for two holes configuration ($\beta = 90^{\circ}$).

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Fig. 10. SCF at the defense hole as a function of α corresponding to different sizes of defense hole for two holes configuration ($\beta = 90^{\circ}$).

Table 1
The maximum and minimum normalized tangential stresses, and the corresponding locations θ (in degrees) for the two equal holes configuration

	σ_θ/σ		$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 1.0$	$\alpha = 4.0$	$\alpha = 10$
$\beta = 0^\circ$	Maximum	Present	6.107 (0.0°)	4.423 (0.0°)	3.264 (0.0°)	3.020 (180.0°)	3.002 (180.0°)
		Haddon	6.106 (0.0°)	4.423 (0.0°)	3.264 (0.0°)	3.020 (180.0°)	3.003 (180.0°)
	Minimum	Present	−0.962 (88.3°)	−0.924 (89.7°)	−0.886 (91.1°)	−0.920 (90.6°)	−0.974 (90.1°)
		Haddon	−0.962 (88.8°)	−0.924 (89.7°)	−0.886 (91.1°)	−0.920 (90.6°)	−0.974 (90.1°)
$\beta = 90^\circ$	Maximum	Present	2.611 (185.6°)	2.619 (185.4°)	2.650 (184.6°)	2.827 (181.3°)	2.948 (180.2°)
		Haddon	2.611 (185.6°)	2.619 (185.4°)	2.650 (184.6°)	2.827 (181.3°)	2.948 (180.2°)
	Minimum	Present	−0.918 (270.0°)	−0.905 (270.0°)	−0.896 (270.0°)	−0.940 (270.0°)	−0.979 (270.0°)
		Haddon	−0.918 (270.0°)	−0.905 (270.0°)	−0.896 (270.0°)	−0.940 (270.0°)	−0.979 (270.0°)

Table 2
The maximum and minimum SCF corresponding to various values of α for β_1 and β_2 with the two equal holes configuration

	Maximum SCF		Minimum SCF		
α	β_1	SCF	β_2	SCF	Range of β (SCF $< k_0$)
$\alpha = 0.2$	22	6.561	90	2.611	80–90
$\alpha = 0.4$	26	5.166	90	2.619	79–90
$\alpha = 1.0$	28	4.018	90	2.650	76–90
$\alpha = 4.0$	33	3.175	90	2.827	68–90
$\alpha = 10$	35	3.037	90	2.948	64–90

Table 3
The maximum and minimum SCF corresponding to various values of α for β_1 and β_2 with the three equal holes configuration

	Main hole					Defense hole				
	Maximum		Minimum			Maximum		Minimum		
α	β_1	SCF	β_2	SCF	Range (SCF $< k_0$)	β_1	SCF	β_2	SCF	Range (SCF $< k_0$)
$\alpha = 0.2$	23	7.642	75	1.997	69–90	24	7.521	90	2.527	82–90
$\alpha = 0.4$	27	5.859	72	2.050	66–90	28	5.779	90	2.540	81–90
$\alpha = 1.0$	29	4.347	90	2.266	59–90	30	4.315	90	2.581	77–90
$\alpha = 4.0$	34	3.263	90	2.655	60–90	33	3.221	90	2.790	68–90
$\alpha = 10$	37	3.063	90	2.896	60–90	35	3.046	90	2.935	64–90

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Fig. 11. The variation of SCF at the main hole as a function of inclination β for different proportionality constants α with the three equal holes configuration.

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Fig. 12. The variation of SCF at the defense hole as a function of inclination β for different proportionality constants α with the three equal holes configuration.

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Fig. 13. SCF at the main hole as a function of α corresponding to different sizes of defense hole for three holes configuration ($\beta = 0^{\circ}$).

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Fig. 14. SCF at the defense hole as a function of α corresponding to different sizes of defense hole for three holes configuration ($\beta = 0^{\circ}$).

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Fig. 15. SCF at the main hole as a function of α corresponding to different sizes of defense hole for three holes configuration ($\beta = 90^\circ$).

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Fig. 16. SCF at the defense hole as a function of α corresponding to different sizes of defense hole for three holes configuration ($\beta = 90^{\circ}$).

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Fig. 17. SCF at the main hole as a function of α corresponding to different sizes of defense hole for five holes configuration.

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Fig. 18. SCF at the defense hole as a function of α corresponding to different sizes of defense hole for five holes configuration.

alternating method is formulated and applied to analyze the perforated problems with multiple holes under arbitrary loading. The interaction effects among holes can be accurately evaluated through very simple iteration procedure even when the holes are very close. This work can further be extended to analyze multiple holes distributed in a finite plate associated with the finite element method or boundary element method, and will be presented in the near future.

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